Worksheet for October 22

Problems marked with an asterisk are to be placed in your math diary.

(1.*) Consider the double integral $\int \int_D xy \ dA$, where $D.$ is the region in the xy-plane bounded by the hyperboles $xy = 4$ and $xy = 1$ and the lines $y = x$ and $y = 4x$.
  (i) Use graphing software to graph the region D.
  (ii) Find the inverse transformation $F(x, y)$ for $G(u, v) = (uv^{-1}, uv)$ and verify that $F(G(u, v)) = (u, v)$ and $G(F(x, y)) = (x, y)$.
  (iii) Use your inverse transformation to find the domain of integration D_0 in the uv-plane.
  (iv) Use the transformation $G(u, v)$ to find $\int \int_D xy \ dA$.

(2.*) Consider $\int \int_{\mathbb{R}^2} (1 + x^2 + y^2)^p \ dA$, where $p \in \mathbb{R}$ is a constant.
  (i) Determine if the integral converges for $p = -2$ by integrating over the disk of radius $a > 0$ centered at the origin, and letting a tend to infinity.
  (ii) For what values of p does the integral converge ? What is the value when it converges?